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Anomalies & tadpoles

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Abstract: We show that massless RR tadpoles in vacuum configurations with open and unoriented strings are always related to anomalies. RR tadpoles arising from sectors of the internal SCFT with non-vanishing Witten index are in one-to-one correspondence with conventional irreducible anomalies. The anomalous content of the remaining RR tadpoles can be disclosed by considering anomalous amplitudes with higher numbers of external legs. We then provide an explicit parametrization of the anomaly polynomial in terms of the boundary reflection coefficients, i.e. one-point functions of massless RR fields on the disk. After factorization of the reducible anomaly, we extract the relevant WZ couplings in the effective lagrangians.

Keywords: Anomalies in Field and String Theories, Superstring Vacua, Conformal Field Models in String Theory, Compactification and String Models.

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Contents

1. Introduction

Anomaly and tadpole vanishing conditions have been recognized for long as intriguingly related constraints in the construction of consistent vacuum configurations with open and unoriented strings. Indeed infinity and anomaly cancellations go hand by hand in the SO(32) type-I theory [1]. Although considerable experimental evidence has been accumulated in favour of a one-to-one correspondence between the two kinds of consistency conditions in models with $\mathcal{N} = (1, 0)$ supersymmetry in $D = 6$, already in $D = 4$ there are exceptions to this rule and very little is known so far in the non-supersymmetric case. At first sight it is not obvious why the vanishing of RR tadpoles, a transverse-channel constraint on the closed-string coupling to boundaries and crosscaps, should in general be related to the vanishing of irreducible anomalies, a constraint on the spectrum that is naturally encoded in the direct-channel amplitudes.

The issue has been first addressed in [2] where the RR tadpole in $D = 10$ was shown to induce a violation of BRST invariance on the string worldsheet. In [4, 3] tadpole cancellation was proposed as the open-string analogue of closed-string modular invariance, that cuts off the UV region responsible of anomalies. Additional evidence for a tadpole-anomaly correspondence was given in [5], where some $\mathcal{N} = (1, 0)$ supersymmetric models in $D = 6$ and some non-supersymmetric models in $D = 10$ were shown to be free of irreducible anomalies thanks to the vanishing of RR tadpoles. The cancellation of the left-over reducible anomalies relied on the presence of extra RR antisymmetric tensors that participate in a generalized Green-Schwarz mechanism [3]. The general belief on an IR-UV correspondence between RR tadpoles and anomalies has been reinforced by the advent of D-branes [7]. RR charge neutrality of vacuum configurations with closed unoriented strings requires the introduction of D-branes and their open-string excitations. The new trend motivated the study of [8], where the correspondence was analyzed for supersymmetric D-brane configurations at orbifold singularities in non-compact spaces. RR tadpoles from twisted sectors were identified with irreducible gauge anomalies. More recently, an important contribution has been given in [9], where a careful comparison has been drawn in the context of geometric (supersymmetric) orbifold compactifications to $D = 4, 6$. In all cases studied in [9], RR tadpoles associated to string amplitudes in sectors where the orbifold group acts without fixed tori have been put in a one-to-one correspondence with irreducible anomalies. The remaining tadpole conditions have been left as seemingly unrelated to the conditions for anomaly cancellation. The intrisic reasons for such a tadpole-anomaly correspondence and the range of its validity inside the more general web of open string compactifications remains unclear.

The increasing interest in the phenomenological perspectives of open string vacuum configurations with various patterns of supersymmetry breaking deserves a thorough analysis of this long-standing problem. In the present paper we address the problem in full generality. Our analysis relies on the topological properties of the internal SCFT (super-conformal field theory) and encompasses not only unoriented descendants of geometric orbifolds and Gepner models, but also descendants of asymmetric orbifolds and free fermionic constructions that result in left-right symmetric parent theories. We find that RR-tadpole conditions always correspond to the cancellation of some sort of anomaly in the effective theory. The contribution to the anomalies from a given sector of the internal SCFT is measured by its "chiral" Witten index, $\mathcal{I} = \mathrm{tr}_R(-)^F$ [10]. This index is defined by a chiral vacuum amplitude in the odd spin structure, where the worldsheet supercurrent is periodic, and effectively counts the chiral asymmetry of each sector.[1] Tadpoles associated to sectors with non-vanishing Witten index will be precisely identified with irreducible terms in the canonical anomaly polynomial. This extends the notion of sectors without fixed tori that appear in the orbifold constructions discussed in [9] to the present more general context. In addition an attentive study of the tadpoles that arise from sectors with vanishing Witten index suggests their connection to anomalous amplitudes involving a larger number of external insertions.

We would like to stress that nowhere in the paper we will assume that the theory enjoys target space supersymmetry. Our discussion is completely general and applies to non-supersymmetric models that can arise for instance from superstring compactifications with or without brane supersymmetry.

[1] For definitions and applications of the index in the open string context see [11].

In order to keep the notation as compact as possible, the language we use is the one of rational superconformal field theories but with obvious modifications one can accomodate in this setting all "irrational" models studied so far such as tori and orbifolds thereof. The only crucial ingredient in our considerations is the worldsheet consistency between direct (loop) and transverse (tree) channel for the relevant amplitudes. This imposes highly non-trivial constraints in the construction of a sensible open string model. Throughout the paper we will assume that a solution to these fundamental constraints (not to be confused with the tadpole-anomaly conditions under consideration) has been found. Another important ingredient is the "modular invariance" of string amplitudes in the odd-spin structure which allows us to relate irreducible anomalies in the direct channel to RR tadpoles in the transverse channel.

After showing the equivalence between RR tadpoles and irreducible terms in the anomaly polynomial, we will pass to the study of the reducible ones. We parametrize the anomaly polynomials in terms of reflection coefficients for the massless closed string states in front of crosscaps and boundaries. We will find that, barring some minor ambiguities in $D = 8, 10$ if the Chan-Paton group is non semi-simple, the anomaly polynomial admits a unique factorization in terms of a sum of as many products as sectors that flow in the transverse channel and have non-vanishing Witten index. This allow us to factorize the reducible anomaly in such a way as to immediately expose the R-R fields that participate in the generalized GSS mechanism and to extract the corresponding terms in the effective lagrangian. In particular one can immediately identify the "anomalous" U(1)'s.

The plan of the paper is as follows. In section 2 we briefly review the construction of open string descendants. In section 3 we establish the precise relation between RR tadpoles for sectors with non-vanishing Witten index and irreducible anomalies. In section 4 we study the reducible part of the anomaly polynomials in any even dimensions and extract the WZ couplings in the effective theories. In section 5 we discuss RR tadpoles arising from sectors with vanishing Witten index and elaborate on their connection to anomalous amplitudes with higher number of external legs. In section 6 we discuss our results and add some concluding remarks.

2. Open string descendants

In this section we review some relevant features in the construction of open string descendants of generic left-right symmetric "compactifications" of type-II and type-0 superstrings. By superstring compactification we mean any vacuum configuration whose worldsheet dynamics is governed by a superconformal field theory (SCFT) irrespective of the presence of unbroken target-space supersymmetry.

A closed string compactification to D dimensions is defined by tensoring an internal SCFT, with left and right moving central charges $(c, \bar{c}) = (\frac{3}{2}(10-D), \frac{3}{2}(10-D))$, with a $(c, \bar{c}) = (\frac{3}{2}D, \frac{3}{2}D)$ spacetime theory realized in terms of free worldsheet bosons and fermions $\{X^\mu, \psi^\mu, \tilde{\psi}^\mu\}$. The Hilbert space of string states can be decomposed into a (generically infinite) sum $\mathcal{H} = \oplus_{i,j}\mathcal{H}_i \otimes \bar{\mathcal{H}}_j$. The index i labels the primary fields Φ^i of some chiral algebra $\mathcal{G}$, that includes an $\mathcal{N}=1$ superconformal algebra at least. The left-moving subspace $\mathcal{H}_i$ thus consists in the tower of descendants under $\mathcal{G}$ of the groundstate $|h_i\rangle = \Phi^i|0\rangle$. The spectrum of the left-moving hamiltonian L_0 in the sector $\mathcal{H}_i$ is conveniently assembled in the holomorphic character

$$\mathcal{X}_i(\tau) = \mathrm{Tr}'_{\mathcal{H}_i}\; q^{L_0 - \frac{c}{24}}\,, \tag{2.1}$$

with $q = e^{2\pi i \tau}$. A prime in (2.1) indicates the omission of the contribution of the bosonic zero-modes to the trace. Moreover, it is always understood that states in the NS (sub)sector, where the worldsheet supercurrent is anti-periodic, enter with a plus sign and states in the R (sub)sector, where the worldsheet supercurrent is periodic, enter with a minus sign. This follows from modular invariance at two loops and implements the correct relation between spin and statistics. For the right-moving excitations on the string worldsheet we assume $\bar{\mathcal{G}} \sim \mathcal{G}$ and define anti-holomorphic characters $\bar{\mathcal{X}}_j(\bar{\tau})$ similarly.

The Lorentz transformation properties of states in $\mathcal{H}_i$ are encoded in the space-time part of the character $\mathcal{X}_i$, that can be decomposed into "characters" of the SO$(D-2)$ little group

$$\begin{aligned}
\chi_O + \chi_V &= \left(\frac{\vartheta\left[{0 \atop 0}\right]}{\eta^3}\right)^{\frac{D-2}{2}} \qquad & \chi_O - \chi_V &= \left(\frac{\vartheta\left[{0 \atop 1/2}\right]}{\eta^3}\right)^{\frac{D-2}{2}} \\
\chi_S + \chi_C &= \left(\frac{\vartheta\left[{1/2 \atop 0}\right]}{\eta^3}\right)^{\frac{D-2}{2}} \qquad & \chi_S - \chi_C &= \left(\frac{-i\vartheta\left[{1/2 \atop 1/2}\right]}{\eta^3}\right)^{\frac{D-2}{2}}\,.
\end{aligned} \tag{2.2}$$

The labels O, V, S, C denotes the scalar, vector, spinor left (L) and spinor right (R) representation of the affine transverse Lorentz algebra at level $\kappa = 1$. Once the contributions of the spacetime bosonic and fermionic coordinates are included, modular invariance ensures that any holomorphic character is given by a sum over spin structures:

$$\mathcal{X}_i = \sum_{\alpha,\beta=0,1/2} \left(\frac{\vartheta\left[{\alpha \atop \beta}\right]}{\eta^3}\right)^{\frac{D-2}{2}} \mathcal{I}_i\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \mathcal{I}_i\Theta_D + \cdots \tag{2.3}$$

and similarly for the antiholomorphic part. In the second equality we have isolated the contribution of the odd spin structure on which our analys will be focussed. In this spin-structure worldsheet bosons and fermions share the same boundary conditions. The worldsheet supercurrent is periodic and the contributions of the massive excitations to the sum cancel against each other. The contribution of the spacetime super-coordinates,

$$\Theta_D = \left(\frac{-i\vartheta\left[\begin{smallmatrix}1/2\\1/2\end{smallmatrix}\right]}{\eta^3} \right)^{\frac{D-2}{2}} , \tag{2.4}$$

though formally zero, is a convenient book-keeping of the chiral asymmetry. The result of the trace in each sector of the internal SCFT theory, $\mathcal{I}_i \equiv \mathcal{I}_i\left[\begin{smallmatrix}1/2\\1/2\end{smallmatrix}\right]$, is an integer (the Witten index [10]) that counts the difference between the number of bosonic and fermionic ground-states. From the right-hand side of (2.3) we notice that $\mathcal{I}_i$ effectively counts the difference between the number of left- (L) and right- (R) "handed" spacetime fermions.

The spectrum of perturbative closed string states is then packaged in the one-loop (torus) partition function. Neglecting overall volume factors and the modular integration ($\int_{\mathcal{F}} d^2\tau/\tau_2^2$) one finds

$$\mathcal{T} = \tau_2^{-\frac{D-2}{2}} \sum_{ij} \mathcal{T}_{ij} \mathcal{X}_i \bar{\mathcal{X}}_j \,. \tag{2.5}$$

$\mathcal{T}_{ij}$ are positive integer coefficients with $\mathcal{T}_{00} = 1$ for the uniqueness of the identity sector, i.e. of the graviton. The powers of τ_2 arise from the momentum integrations in the non-compact directions. The condition for left-right symmetry on the worldsheet translates into the constraints $\mathcal{T}_{ij} = \mathcal{T}_{ji}$. The coefficients $\mathcal{T}_{ij}$ are also highly restricted by one-loop modular invariance. The characters are known to provide a unitary representation of the modular group $\mathrm{SL}(2,\mathbb{Z})$ generated by the transformations T and S under which

$$\begin{aligned} T: &\quad \mathcal{X}_i(\tau+1) = e^{2\pi i(h_i - c/24)} \mathcal{X}_i(\tau) \\ S: &\quad \mathcal{X}_i\left(-\frac{1}{\tau}\right) = \sum_j (i\tau)^{-\frac{D-2}{2}} S_{ij} \mathcal{X}_j(\tau) \,. \end{aligned} \tag{2.6}$$

After the resolution of fixed-point ambiguities, related to the presence of different sectors with the same character, the transformation S is represented by a symmetric matrix that satisfies $(ST)^3 = S^2 = C$, with C the charge conjugation matrix.

In most of our subsequent discussion, we will restrict our attention to the case in which the boundary conditions associated to the introduction of open and unoriented strings preserve the diagonal combination of the worldsheet symmetries of the bulk theory together with their target-space byproducts. This generalizes the notion of a BPS D-brane. With a slight change of notation, that amounts to decomposing the characters of the parent theory into their irreducible components with respect to some lower (super-)symmetry, one can easily accomodate models with brane (super-)symmetry breaking. This generalizes the notion of a non-BPS D-brane. Moreover we will display formulas which are more akin to the rational context, where the number of characters is finite. With some additional care, the results can be adapted to irrational contexts such as toroidal or orbifold compactifications at irrational values of the moduli (radii, shapes and Wilson lines).

At generic points of the moduli space of toroidal compactifications [26] the index i may be thought to run over an infinite number of primary fields, one for each independent choice of internal momenta and windings. Similarly in orbifold compactifications, only states that are invariant under the action of the orbifold group will enter the trace (2.1). Chiral string excitations are then organized according to their eigenvalues under the action of the orbifold group. Taking $\mathbb{Z}_N$ for simplicity, one is lead to define the characters [12]

$$\mathcal{X}_{gh} = \frac{1}{N} \sum_{k=0}^{N-1} \rho_{gk} \, \omega_h^k \, , \tag{2.7}$$

with $g, h = 0, 1, \ldots, N-1$ labelling all possible twists in the σ and τ directions and $\omega_h = e^{2\pi i h/N}$. In the unwisted sectors and in sectors with fixed tori, there are infinitely many characters depending on the choices of allowed momenta and windings. In twisted sectors without fixed tori, a finite number of twisted characters lives at each fixed point. The chiral amplitudes ρ_{gh} are defined by the traces

$$\rho_{gh} = \mathrm{Tr}_g \; h \, q^{L_0 - c/24} \, . \tag{2.8}$$

in the g-twisted sector. Explicit expressions for these traces in $D = 4, 6$ can be found in the [12, appendix A].

By suitably tuning the parameters, any toroidal or orbifold compactification can be described in terms of a rational SCFT. Moreover in both the rational and the irrational contexts the number of massless characters, i.e. sectors with massless ground-states, is finite. Since only these states will be relevant to our analysis the two cases can be treated in parallel. Focussing on the odd spin-structure, where anomalies potentially reside, one is effectively dealing with the "topological" part of the theory that is largely independent of the moduli of the SCFT. For instance, only massless sectors with non-vanishing Witten index enter the computation of the Euler number of the "compactification manifold" M [10, 13],

$$\chi(M) = \sum_{i,j} \mathcal{T}_{ij} \mathcal{I}_i \mathcal{I}_j \, , \tag{2.9}$$

and the other curvature invariants that allow one to identify the topological class the vacuum configuration belongs to. On the other hand, massive string states do not contribute to the Witten index since they always come in Bose-Fermi degenerate pairs. Ground-states that can become massless at special points (e.g. decompactification limit) of the moduli space, though not contributing to the naïve Witten index, enter the computation of anomalous amplitudes in a subtle way.

Let us now consider the unoriented descendant of a left-right symmetric closed-string theory, the "parent" theory, defined by (2.5). For definiteness we will consider orientifold reductions by the worldsheet parity operator Ω. More general unoriented descendants that combine Ω with other internal symmetries can be similarly studied. The unoriented closed string spectrum is determined by halving (2.5) and adding to it the Klein-bottle projection. Up to overall volume factors and the modular integration ($\int dt/t$) one has

$$\mathcal{K} = \frac{1}{2}\,\mathrm{Tr}_{\mathrm{closed}}\,(\Omega\, q^{L_0-\frac{c}{24}}) = \frac{1}{2\,t^{D/2}}\sum_i K^i \mathcal{X}_i(2it) = \frac{1}{2\,t^{D/2}}\sum_i K^i \mathcal{I}_i \Theta_D(2it) + \cdots \tag{2.10}$$

In the right hand side, for later convenience, only the contribution of the odd spin-structure Θ_D has been displayed. The sum in (2.10) is restricted to the diagonal terms ($\mathcal{T}_{ii} \neq 0$) in (2.5). For permutation modular invariants ($\mathcal{T}_{ij} = 0, 1$) the Klein-bottle coefficients K^i, implementing the action of Ω on $\mathcal{H}_i \otimes \bar{\mathcal{H}}_i$, are related to the ones in the torus partition function through $K^i = \pm \mathcal{T}_{ii}$.

Whenever the action of Ω is not free the consistency of the theory requires the inclusion of boundaries and the corresponding open string sectors ending on them. The open string partition function consists in the Annulus and Möbius-strip amplitudes. Neglecting the overall volume factors and the modular integration ($\int \frac{dt}{t}$) one has

$$\begin{aligned}
\mathcal{A} &= \frac{1}{2\,t^{D/2}}\sum_{i,a,b} A^i_{ab} n^a n^b \mathcal{X}_i\left(\frac{it}{2}\right) = \frac{1}{2\,t^{D/2}}\sum_{i,a,b} A^i_{ab} n^a n^b \mathcal{I}_i \Theta_D\left(\frac{it}{2}\right) + \dots . \\
\mathcal{M} &= \frac{1}{2\,t^{D/2}}\sum_{i,a} M^i_a n^a \widehat{\mathcal{X}}_i\left(\frac{it}{2}+\frac{1}{2}\right) = \frac{1}{2\,t^{D/2}}\sum_{i,a} \mathcal{I}_i M^i_a n^a \widehat{\Theta}_D\left(\frac{it}{2}+\frac{1}{2}\right) .
\end{aligned} \tag{2.11}$$

A proper basis of real hatted characters $\widehat{\mathcal{X}}_i = T^{-1/2}\mathcal{X}_i$ has been introduced in $\mathcal{M}$ [5]. The indices a, b run over the number of boundaries, i.e. independent Chan-Paton charges, with integer multiplicities n^a. Barring some exceptions to be discussed later, the number of independent charges one should introduce equals the number of characters $\mathcal{X}_i$ that are paired with their charge conjugates $\bar{\mathcal{X}}_{iC}$ in the parent theory (2.5). The integers A^i_{ab} and M^i_a count the number of times the sector i runs in an Annulus and Möbius-strip loop, respectively, of open strings with ends on (a, b) and (a, a) respectively. Here and in the following, we will not distinguish between "complex" (unitary) and "real" (orthogonal or symplectic) Chan-Paton multiplicities n^a. The sum over a will include two contributions for the former and only one for the latter, thus reproducing not only the correct dimensions of the representations but also the correct orientation of the boundary.

The quantities appearing in (2.10) and (2.11) are highly constrained by two consistency requirements.

The first is the requirement of a consistent interpretation of the transverse channel of these amplitudes as the tree-level exchange of closed string states between boundaries and/or crosscaps. This implies that under $t \to 1/t$ the coefficients of the above three amplitudes should reconstruct perfect squares

$$\begin{aligned}
\widetilde{\mathcal{K}} &= \frac{2^{D/2}}{2}\sum_i (\Gamma^i)^2 \mathcal{X}_i(q) = \frac{2^{D/2}}{2}\sum_i (\Gamma_i)^2 \mathcal{I}_i \Theta_D(q) + \cdots \\
\widetilde{\mathcal{A}} &= \frac{2^{-D/2}}{2}\sum_{i,a} (B^i_a n^a)^2 \mathcal{X}_i(q) = \frac{2^{-D/2}}{2}\sum_{i,a} (B^i_a n^a)^2 \mathcal{I}_i \Theta_D(q) + \cdots \\
\widetilde{\mathcal{M}} &= \frac{2}{2}\sum_{i,a} (\Gamma^i B^i_a n^a) \widehat{\mathcal{X}}_i(-q) = -\frac{2}{2}\sum_{i,a} (\Gamma^i B^i_a n^a) \mathcal{I}_i \widehat{\Theta}_D(-q) + \cdots
\end{aligned} \tag{2.12}$$

The relative powers of 2 result from the different rescalings of the modular parameters ($\tau_{\mathcal{K}} = 2it$, $\tau_{\mathcal{A}} = it/2$, $\tau_{\mathcal{M}} = it/2 + 1/2$) that naturally enter the definition of the amplitudes in the direct channel. These rescalings are necessary in order for the amplitudes in the transverse channel to be expressed in terms of the common length of the tube $\ell = -\frac{1}{2\pi}\log q$. The coefficients Γ^i and B^i_a should then be interpreted as the reflection coefficient (one-point function) on a crosscap and on a boundary of type a respectively. They are related to the integer coefficients in the direct channel by suitable modular transformations:

$$\begin{aligned}
K^i &= \sum_j S^i{}_j \Gamma^j \Gamma^j \\
A^i_{ab} &= \sum_j S^i{}_j B^j_a B^j_b \\
M^i_a &= \sum_j P^i{}_j \Gamma^j B^j_a \,,
\end{aligned} \tag{2.13}$$

with S defined in (2.6) and $P \equiv T^{1/2} S T^2 S T^{1/2}$.

In addition one should require that fluxes of massless RR-fields should not be trapped inside the "compactification" manifold. This leads to the tadpole cancellation conditions:

$$2^{D/2}\Gamma^i + \sum B^i_a n^a = 0 \,, \tag{2.14}$$

where i runs over any sector that flow in the transverse channel and contain RR massless states. Understanding how to precisely relate these conditions to another consistency requirement, the absence of irreducible anomalies in the low-energy field theory, will be the main subject of our investigation.

Finding a solution to (2.13) is in general a highly non-trivial step in the construction of an open descendant. We will always assume that this has been properly done. It is worth stressing that a "canonical" solution always exists if the torus modular invariant is given by the charge conjugation matrix $\mathcal{T}_{ij} = C_{ij}$ ("geometric compactifications" such as tori, symmetric orbifolds[2] and Gepner models belong in

[2]As we will see, in some orbifold models this may require some identifications among the Chan-Paton charges.

this category). In this case the number of independent Chan-Paton charges is exactly equal to the number of characters and one can use the same kind of indices, say $i, j, \ldots$, to label both. The ansatz [14, 5, 15] amounts to taking

$$\begin{aligned} K^i &= {Y_{00}}^i \\ {A_{ij}}^k &= {N_{ij}}^k \\ {M_j}^i &= {Y_{0j}}^i \,, \end{aligned} \tag{2.15}$$

where

$${N_{ij}}^k = \sum_l \frac{S_{il} S_{jl} (S^\dagger)^{lk}}{S_{0l}} \tag{2.16}$$

are the fusion rule coefficients, that compute how many independent couplings of the primary fields i and j give the primary field k, and

$${Y_{ij}}^k = \sum_l \frac{S_{il} P_{jl} (P^\dagger)^{lk}}{S_{0l}} \tag{2.17}$$

are (not necessarily positive) integers that satisfy ${Y_{ij}}^k = {N_{ij}}^k \bmod 2)$. The boundary and crosscap reflection coefficients then read

$$\begin{aligned} \Gamma^i &= \frac{P^i{}_0}{\sqrt{S_{0i}}} \\ B^i{}_j &= \frac{S^i{}_j}{\sqrt{S_{0i}}} \,. \end{aligned} \tag{2.18}$$

Notice that quantities in the transverse amplitudes (2.12) are related to the spectra (2.10) and (2.11) through the S and P modular transformations, whose details vary from model to model. Fortunately this is not the case for the contribution of the odd-spin structure in (2.3). Each sector of the internal SCFT only enters through its Witten index $\mathcal{I}_i$ that is modular invariant and the modular transformations of Θ_D are simply

$$\begin{aligned} \Theta_D(-1/it) &= (it)^{\frac{D-2}{2}} \Theta_D(it) \\ \Theta_D(it+1) &= \Theta_D(it) \,. \end{aligned} \tag{2.19}$$

This elementary observation will be the basis of all our manipulations in what follows.

3. Anomalies in open string descendants

In this section we discuss the relation between RR tadpoles and irreducible anomalies in a generic even-dimensional open descendant. We first review (and slightly adapt) the results of [16, 17], which allow us to reproduce the anomaly polynomial by an string computation in the unoriented descendant. Our strategy will be to use modular invariance of the odd-spin structure (2.19) to parametrize the anomaly polynomial in terms of the reflection coefficients in the transverse channel. On the one hand, this allows us to identify the precise combination of RR tadpoles that corresponds to the irreducible gravitational anomaly. On the other hand, using the completeness properties of the boundary reflection coefficients, this will enable us to map RR tadpoles in sectors with non-vanishing Witten index to irreducible gauge anomalies. After imposing the tadpole/anomaly conditions, the anomaly polynomial turns out to be reducible. It thus admits a simple factorization that suggests the participation of various RR antisymmetric tensors of different rank to a generalized mechanism of anomaly cancellation.

Anomalies are encoded in the odd-spin structure part of the one-loop string amplitudes

$$\left\langle \prod_{f=1}^{N} V_F(p_f,\xi_f) \prod_{g=1}^{M} V_G(p_g,h_g) \right\rangle \tag{3.1}$$

involving N gauge field V_F and M graviton V_G vertex operators with polarization and momenta ξ_f, p_f and h_g, p_g, respectively. The number of external legs is such that $N+M = D/2+1$, where D is the (even) number of non-compact dimensions. One of the vertices has to be taken with a longitudinal polarization. Since the parent theory is assumed to be anomaly free[3] anomalous contributions to the above amplitudes only arise from the Klein bottle, Annulus and Möbius strip. In the odd spin-structure there is one supermodulus (zero-mode of the spin 3/2 bosonic superghost β) and one conformal Killing spinor (zero-mode of the spin $-1/2$ bosonic superghost γ). In order to dispose of the former a picture changing operator is to be inserted. In order to dispose of the latter, one of the vertices (let's say the one with longitudinal polarization) should be taken in the (-1) picture. As a result the total superghost charge remains zero as expected for surfaces with vanishing Euler characteristic. After simple manipulations (see [17] for details) a generating function for the anomalous amplitudes can be written as

$$A = \int_0^\infty dt \frac{d}{dt} \langle e^{-S_0+S_F+S_R} \rangle_{\mathrm{odd}} \tag{3.2}$$

where S_0 is the free action, and the exponentiated effective vertex operators S_F, S_R are given by

$$\begin{aligned} S_F &= \oint ds\, F^a \lambda^a \\ S_G &= \int d^2z\, R_{\mu\nu} \left[X^\mu (\partial + \bar\partial) X^\nu + \left(\psi^\mu - \tilde\psi^\mu\right)\left(\psi^\nu - \tilde\psi^\nu\right) \right], \end{aligned} \tag{3.3}$$

with

$$F^a = \frac{1}{2} F^a_{\mu\nu} (\psi_0^\mu \psi_0^\nu)\,, \qquad R_{\mu\nu} = \frac{1}{2} R_{\mu\nu\rho\sigma} (\psi_0^\rho \psi_0^\sigma)\,. \tag{3.4}$$

bilinears in the zero modes of non-compact fermionic coordinates. After integration over the grassmanian variables, F^a and $R_{\mu\nu}$ behave as two-forms.

[3] Modular invariance ensures that this is always the case.

The above determinant has been computed in [18] (see also [19] for additional insights). The crucial point is that the determinant, being defined by a trace in the odd-spin structure where worldsheet bosons and fermions share the same boundary conditions, is given by a t-independent topological invariant. Each sector of the internal SCFT only enters through its Witten index $\mathcal{I}_i$ and the final result can be written as [18]

$$\begin{aligned}
\mathcal{K}_{\rm odd} &= -\frac{1}{2}\sum_i \mathcal{I}_i K^i\, I_A(R) \\
\mathcal{A}_{\rm odd} &= = \frac{1}{4}\sum_{i,a,b} \mathcal{I}_i A^i_{ab}\, {\rm ch}_{n^a}(F)\, {\rm ch}_{n^b}(F) I_{1/2}(R) \\
\mathcal{M}_{\rm odd} &= \frac{1}{4}\sum_{i,a} \mathcal{I}_i M^i_a n^a\, {\rm ch}_{n^a}(2F) I_{1/2}(R)\,,
\end{aligned} \tag{3.5}$$

where ${\rm ch}(F)$ is the Chern character and $I_A(R)$ and $I_{1/2}(R)$ represent the contributions to the gravitational anomaly of a self-dual antisymmetric tensor and a complex spin 1/2 L-fermion respectively. The additional factor of one-half in the Annulus and Möbius-strip amplitudes reflects the fact that they are counting real fermions. The relation between spin and statistics is responsabile for the extra minus sign of the Klein-bottle contribution with respect to the Annulus and Möbius strip. The former can only contribute loops of (anti)self-dual antisymmetric tensors while the latter can only contribute fermionic loops. In terms of the Hirzebruch polynomial $\widehat{\mathcal{L}}(R)$ and A-roof genus $\widehat{\mathcal{A}}(R)$ one has

$$\begin{aligned}
I_A(R) &= \frac{1}{8}\widehat{\mathcal{L}}(R) = -\frac{1}{8} + \frac{1}{48}R^2 + \frac{1}{32}\left(\frac{7}{45}R^4 - \frac{1}{9}(R^2)^2\right) + \\
&\quad + \frac{1}{128}\left(\frac{496}{2835}R^6 - \frac{588}{2835}R^2R^4 + \frac{140}{2835}(R^2)^3\right) + \cdots \\
I_{1/2}(R) &= \widehat{\mathcal{A}}(R) = 1 + \frac{1}{48}R^2 + \frac{1}{32}\left(\frac{1}{180}R^4 + \frac{1}{72}(R^2)^2\right) + \\
&\quad + \frac{1}{128}\left(\frac{1}{2835}R^6 + \frac{1}{1080}R^2R^4 + \frac{1}{1296}(R^2)^3\right) + \cdots \\
{\rm ch}_{n^a}(F) &= {\rm tr}_{n^a}(\exp iF) \equiv \sum_k \frac{1}{k!}F^k_a\,,
\end{aligned} \tag{3.6}$$

where by R^{2m} we mean ${\rm tr}_V\, R^{2m}$ and wedge products are always understood. In the last line we have introduced the shorthand notation $F^k_a = {\rm tr}_{n^a}\, F^k$ for later convenience.

The absence of anomalies in the parent theory allows one to rewrite the Klein-bottle contribution in terms of the spectrum of massless closed-string states of the unoriented descendant. As in [18], we find the expected field-theory result

$$\mathcal{K} = -\frac{1}{2}\sum_i \mathcal{I}_i K_i I_A(R) = (n^L_A - n^R_A) I_A(R) + (n^L_{3/2} - n^R_{3/2}) I_{3/2}(R) + (n^S_{1/2} - n^C_{1/2}) I_{1/2}(R)\,, \tag{3.7}$$

where $n_A^{L,R}$, $n_{1/2}^{L,R}$ and $n_{3/2}^{L,R}$ are respectively the numbers of antisymmetric tensors, spin 1/2 and spin 3/2 massless closed-string states with definite chirality properties and and $I_{3/2}(R) = (\mathrm{ch}_V(R) - 1)\widehat{\mathcal{A}}(R)$. Similarly the contribution to the anomaly polynomial coming from the Annulus and Möbius-strip amplitudes (3.5) reproduces the expected field theory result. Piecing everything together yields

$$\mathcal{P}(R,F) = \sum_i \mathcal{I}_i \left[-\frac{1}{2} K^i I_A(R) + \frac{1}{4} \left(\sum_{a,b} A^i_{ab} \,\mathrm{ch}_{n^a}(F)\,\mathrm{ch}_{n^b}(F) + \right.\right.$$
$$\left.\left. + \sum_a M^i_a \,\mathrm{ch}_{n^a}(2F) \right) I_{1/2}(R) \right] . \qquad (3.8)$$

In (3.8) the trace in the (anti-)symmetric representation of the gauge group a is written in terms of the trace in the n_a-dimensional fundamental representation by means of

$$\mathrm{ch}_{\frac{1}{2}n^a(n^a\pm1)}(F) = \frac{1}{2}\left[\mathrm{ch}_{n^a}(F)^2 \pm \mathrm{ch}_{n^a}(2F)\right] . \qquad (3.9)$$

Few remarks are in order. First, modular invariance of the parent closed string theory not only implies the vanishing of the irreducible anomalies but also of reducible ones, i.e. no GS mechanism is at work in the parent theory. This is well-known for left-right symmetric compactifications but we suspect it to be true in any perturbative vacuum configuration. Second, the Klein bottle only contributes to the pure gravitational anomaly. We have not bother to turn on the field-strengths of any closed-string vector boson since all massless fermions of a left-right symmetric closed-string theory are neutral with respect to them. It is well-known that RR vector bosons are not minimally couple to perturbative string states and that non-abelian NS-NS vector bosons may be present in left-right symmetric theories only if not supersymmetric. Finally the CPT theorem in $D = 4k$ dimensions requires $n_A^L = n_A^R$, $n_{3/2}^L = n_{3/2}^R$ and $n_{1/2}^L = n_{1/2}^R$. As a consequence closed string states do not contribute to the anomaly polynomial in these dimensions. The Klein bottle may give a non-trivial contribution only in $D = 4k + 2$ dimensions. By the same token, the open string sector may contribute to the anomaly in $D = 4k$ dimensions only when the massless fermions are chiral and belong to complex representations of the Chan-Paton group. Since spinorial representations of orthogonal groups cannot appear perturbatively and symplectic groups only admit (pseudo-)real representations, only when unitary groups are present with associated "complex" Chan-Paton charges can the theory be chiral in $D = 4k$ dimensions. In $D = 4k + 2$ dimensions, on the contrary, barring few exceptions, any unpaired massless fermion contributes to gauge, gravitational and mixed anomalies.

We are now ready to pin down the UV-IR correspondence between anomalies and tadpoles in open descendants. Using the general relations (2.13) and the fact that the Witten index, being a t-independent integer number is invariant under modular transformations, i.e.

$$\sum_i \mathcal{I}_i {S^i}_j = \mathcal{I}_j$$
$$\sum_i \mathcal{I}_i {P^i}_j = \mathcal{I}_j\,, \tag{3.10}$$

one can freely transfer the information encoded in the massless sectors with non-vanishing Witten index from one channel to the other. Indeed, saturing (2.13) with $\mathcal{I}_i$ and using (3.10) one can establish the much simpler dictionary

$$\begin{aligned}\sum_i \mathcal{I}_i K^i &= \sum_i \mathcal{I}_i \Gamma^i \Gamma^i \\ \sum_i \mathcal{I}_i A^i_{ab} &= \sum_i \mathcal{I}_i B^i_a B^i_b \\ \sum_i \mathcal{I}_i M^i_a &= \sum_i \mathcal{I}_i B^i_a \Gamma^i\,,\end{aligned} \tag{3.11}$$

that does not explicitly involve any modular transformation. Plugging these relations into the anomaly polynomial (3.8) yields

$$\begin{aligned}\mathcal{P}(R,F) = \sum_i \mathcal{I}_i \Bigg[&-\frac{1}{2}(\Gamma^i)^2 I_A(R) + \frac{1}{4}\left(\sum_a B^i_a \,\mathrm{ch}_{n^a}(F)\right)^2 I_{1/2}(R) + \\ &+\frac{1}{4}\Gamma^i \sum_a B^i_a \,\mathrm{ch}_{n^a}(2F) I_{1/2}(R)\Bigg]\,.\end{aligned} \tag{3.12}$$

The consistency of the effective theory requires the cancellation of the irreducible terms in (3.12). In particular, for the pure gravitational anomaly, using the relation

$$I^D_A(R)|_{\mathrm{irr}} = \frac{2^{D/2}(2^{D/2}-1)}{2} I^D_{1/2}(R)\Big|_{\mathrm{irr}}\,, \tag{3.13}$$

the vanishing of the coefficient of $\mathrm{tr}_V\, R^{D/2+1}$ in the expansion of (3.8) requires

$$\sum_i \mathcal{I}_i \left(2^{D/2}\Gamma^i + \sum_a B^i_a n^a\right)\left((2^{D/2}-1)\Gamma^i + \sum_a B^i_a n^a\right) = 0\,. \tag{3.14}$$

Notice that this is a specific linear combination of the RR tadpole conditions (2.14). In particular the cancellation of the tadpoles automatically implies the cancellation of this irreducible anomaly.

Similarly the absence of irreducible gauge anomaly, i.e. the terms $\mathrm{tr}_{n^b}\, F^{D/2+1}$ in the expansion of (3.8), is equivalent to the conditions[4]

$$\sum_i \mathcal{I}_i \left(2^{D/2}\Gamma^i + \sum_a B^i_a n^a\right) B^i_b = 0\,. \tag{3.15}$$

[4]Notice that U(n) field strengths couple to n and $\bar{n}$ complex charges, and therefore the corresponding gauge anomalies (both irreducible and reducible) will always include the sum of two terms involving traces in the n and $\bar{n}$ after using (3.9).

As apparent one has in principle one tadpole condition for each factor in the Chan-Paton group. Since the number of independent boundary conditions coincides with the number of independent Chan-Paton multiplicities, one may suspect to be on the right track. Indeed one can turn an index a, that labels the factors in the Chan-Paton group, into an index i, that labels the characters that flow in the transverse channel, by making use of the completeness relation, [20]

$$\sum_a B_a^i B_a^j = \frac{\Pi^{ij}}{\sqrt{S_{0i} S_{0j}}}\,, \tag{3.16}$$

valid for any permutation modular invariant parent theory. Π^{ij} is the projector onto the set of characters that flow in the transverse channel and S_{0i} are the elements in the first row/column of the matrix S_{ij} representing modular S-transformations (2.6). S_{0i} are always positive and are sometimes called "quantum dimensions".

By saturating the Chan-Paton index b in (3.15) with B_b^j we are left with

$$\mathcal{I}_i \left(2^{D/2}\Gamma^i + B_a^i n^a\right) \Pi^{ij} = 0\,. \tag{3.17}$$

This precisely reproduces all tadpole cancellation conditions for massless R-R closed string states flowing in the transverse channel (as the presence of Π^{ij} indicates) and belonging to sectors with non-vanishing Witten index $\mathcal{I}_i \neq 0$. It is amusing to observe that in a consistent open descendant in $D = 4k+2$ the absence of irreducible gauge anomalies always implies the absence of irreducible gravitational anomalies. The vanishing of the first factor in (2.14) is selected as the relevant tadpole condition.

A comment is in order. As the attentive reader might have observed, a rephrasing of the above conclusion is necessary in the context of orbifolds. Indeed, in these constructions the worldsheet consistency (2.13) between the direct (one-loop) and transverse (tree) channel often restricts the number of allowed CP charges. As a result the index a often runs over a smaller subset than the set of characters flowing in the transverse channel. If this is the case, the index i in the completeness conditions (3.16) and tadpoles (3.14) and (3.15) should be understood as running over an smaller subset defined by the linear combination of characters flowing in this channel. In the new basis (usually conveniently written in terms of the chiral amplitudes) additional "sectors" with vanishing Witten index arise and the corresponding tadpole information is consequently lost in (3.15).

Modulo this subtlety, we conclude that any massless RR tadpole arising from a sector with non-vanishing Witten index can be identified with some irreducible anomaly. In particular string theory defines a notion of irreducible gauge anomalies even for non-semisimple groups as the ones originating from amplitudes in which all the vertex operators are inserted on the same boundary. Let us now concentrate on the left over reducible part of the anomaly polynomial (3.12).

4. Anomaly polynomial and WZ couplings

Once the tadpole conditions are imposed the anomaly polynomial $\mathcal{P}(R,F)$ is reducible and can be factorized into a sum of products. This factorization allows one to extract the WZ and anomalous couplings of the R-R massless states that participate in a generalized GSS mechanism of anomaly cancellation [6][5] Since the factorization is somewhat sensitive to the (even) dimension of the non-compact part of the target space it is convenient to decompose the total anomaly polynomial in terms of its D-dimensional components

$$\mathcal{P}(R,F) = \sum_D \mathcal{P}_D(R,F)\,, \tag{4.1}$$

where $\mathcal{P}_D(R,F)$, the anomaly polynomial in D-dimension, is a $(D+2)$-form.

After some simple algebra one finds that the factorization is almost always unique. Only in $D=8$ and $D=10$ we find mild ambiguities. The anomaly polynomials in D dimensions explicitly factorize as

$$\begin{aligned}
\mathcal{P}_0(R,F) &= 0\\
\mathcal{P}_2(R,F) &= \frac{1}{4}\sum_{i,a,b}\mathcal{I}_i B^i_a B^i_b\left[X^a_2 X^b_2\right]\\
\mathcal{P}_4(R,F) &= \frac{1}{4}\sum_{i,a,b}\mathcal{I}_i B^i_a B^i_b\left[2X^a_2 X^b_4\right]\\
\mathcal{P}_6(R,F) &= \frac{1}{4}\sum_{i,a,b}\mathcal{I}_i B^i_a B^i_b\left[X^a_4 X^b_4 + 2X^a_2 X^b_6(1)\right]\\
\mathcal{P}_8(R,F) &= \frac{1}{4}\sum_{i,a,b}\mathcal{I}_i B^i_a B^i_b\left[2X^a_4 X^b_6(\xi_b) + 2X^a_2 X^b_8(\xi_b)\right]\\
\mathcal{P}_{10}(R,F) &= \frac{1}{4}\sum_{i,a,b}\mathcal{I}_i B^i_a B^i_b\left[2X^a_2 X^b_{10}(\xi_b) + 2X^a_4 X^b_8(1) + X^a_6(\xi_a) X^b_6(\xi_b)\right]\,,
\end{aligned} \tag{4.2}$$

where, in order to keep the above formulas as compact as possible, we have introduced the definitions

$$\begin{aligned}
X^a_2 &= iF_a\\
X^a_4 &= \frac{i^2}{2!}\left(F^2_a - \frac{n^a}{32}R^2\right)\\
X^a_6(\xi_a) &= \frac{i^3}{3!}\left(F^3_a - \frac{\xi_a}{16}F_a R^2\right)\\
X^a_8(\xi_a) &= \frac{i^4}{4!}\left(F^4_a + \frac{\xi_a-2}{8}F^2_a R^2 + \frac{5-4\xi_a}{1024}n^a (R^2)^2 + \frac{1}{256}n^a R^4\right)
\end{aligned}$$

[5] Explicit examples of WZ lagrangians in supersymmetric $D=4,6$ open string compactifications have been worked it out recently in [17, 23].

$$X^a_{10}(\xi_a) = \frac{i^5}{5!}\left(F^5_a + \frac{5\xi_a - 10}{24}F^3_a R^2 + \frac{10-5\xi^2_a}{768}F_a(R^2)^2 + \frac{1}{96}F_a R^4\right). \qquad (4.3)$$

As mentioned above, the only ambiguities we find are parametrized by real constants ξ_a that enter the definitions of the factors X^a_6, X^a_8 and X^a_{10} in the anomaly polynomials for $D = 8, 10$. Previous analyses [22] correspond to the choice $\xi_a = 1$. The parameters ξ_a could be genuine free parameters in the effective lagrangian or an artifact of the procedure. In order to settle this issue one should compute additional string amplitudes on the disk. It is remarkable that these ambiguities are absent in the supersymmetric case and whenever $\mathrm{tr}_{n^a} F = 0$ for any a, since all ξ_a would disappear from the above formulas.

The nice feature of (4.3) is that, one can easily recognize the closed string RR fields participating in the GSS mechanism of anomay cancellation [1, 6]. Indeed the index i labels the character[6] to which the corresponding closed string field belongs while B^i_a measures the strength of the coupling of a "generalized brane" of type a to these massless R-R states and their massive (super-)partners. Each sector includes $p+1$-form potentials of various degrees. The generalized GSS mechanism requires that a $p+1$-form potential C^i_{p+1} in the sector labelled by i couple with strength B^i_a to a $D-p-1$-form X^a_{D-p-1} of type a in the factorized anomaly polynomial. We can then extract from (4.3) all WZ couplings responsabile for this GSS mechanism in the effective D-dimensional theory

$$\begin{aligned}
L_0 &= 0 \\
L_2 &= \frac{1}{2}\sum_{i,a} B^i_a C^i_0 X^a_2 \\
L_4 &= \frac{1}{2}\sum_{i,a} B^i_a \left(C^i_0 X^a_4 + C^i_2 X^a_2\right) \\
L_6 &= \frac{1}{2}\sum_{i,a} B^i_a \left(C^i_0 X^a_6(1) + C^i_2 X^a_4 + C^i_4 X^a_2\right) \\
L_8 &= \frac{1}{2}\sum_{i,a} B^i_a \left(C^i_0 X^a_8(\xi) + C^i_2 X^a_6(\xi) + C^i_4 X^a_4 + C^i_6 X^a_2\right) \\
L_{10} &= \frac{1}{2}\sum_{i,a} B^i_a \left(C^i_0 X^a_{10}(\xi) + C^i_2 X^a_8(1) + C^i_4 X^a_6(\xi) + C^i_6 X^a_4 + C^i_8 X^a_2\right). \qquad (4.4)
\end{aligned}$$

The factorization of the transverse channel amplitudes in the odd spin structure gives rise to the "odd" propagator which turns a $(p+1)$-form into its dual $(D-p-3)$-form. More precisely

$$\langle dC^i_{p+1} dC^j_{D-p-3}\rangle = \mathcal{I}_i \Pi^{ij} \mathcal{V}_D\,, \qquad (4.5)$$

[6]More precisely the index i refers to the pair (i, i^C) of characters that are paired in the torus partition function.

with $\mathcal{V}_D$ a regularized D-dimensional volume. Notice that one can simultaneously rescale any C^i_{p+1} by a factor α^i_{p+1} and its dual C^i_{D-p-1} by a factor $1/\alpha^i_{p+1}$ without modifying the factorization of the anomaly polynomial. The couplings (4.4) will of course absorb these renormalizations.

From the expression of (4.4) one can easily recognize the anomalous U(1)'s. They are to be identified as the abelian factors of type a entering in X^a_2 and coupling anomalously to C^i_{D-2} with a non-vanishing reflection coefficient B^i_a.

Although we have concentrated our attention on the CP-odd part of the effective lagrangian, in supersymmetric models some interesting CP-even coupling are related to (4.4) by supersymmetry. WZ lagrangians in supersymmetric $D = 4, 6$ open string models have been worked it out in [17, 23]. The terms displyed in (4.4) cannot be the whole story. RR fields belonging to sectors with vanishing Witten index are not present. In particular the WZ couplings RR axion belonging to the identity sector in any supersymmetric vacuum configurations in $D = 4$ are not reproduced by (4.4). In order to get around this problem one has to find a way to disclose the anomalous content encoded in RR tadpoles of sectors with vanishing Witten index.

5. Tadpoles with $\mathcal{I} = 0$ and anomalous amplitudes

In section 3 we have shown that RR tadpoles in sectors with non-vanishing Witten index are in one-to-one correspondence with irreducible anomalies. Unfortunately the vanishing of the Witten index $\mathcal{I}_i$ in a given internal sector makes the associated anomaly condition (3.15) empty and the corresponding tadpole information lost. In this section we argue that a careful look into the anomaly structure of the effective theory may overcome this apparent asymmetry. This will be done at the price of considering anomalous amplitudes involving non-trivial insertions in the internal SCFT. The correct choice is given by the minimal number of insertions that removes the Bose-Fermi degeneracy in a given sector with $\mathcal{I} = 0$. As a result one can once again translate the associated tadpole condition into a condition for anomaly cancellation through (3.15).

Rather than being general let us illustrate this point in the simplest context of even-dimensional ($d = 2\ell = 10 - D$) toroidal compactifications of the SO(32) type-I theory. To start with, we will neither turn on Wilson lines nor expectation values of the $B_{\text{NS-NS}}$-field (equivalent to non-commuting Wilson lines) [26]. We will include their effects in the analysis later on. In this simple situation we have a single massless character $\mathcal{X}_0$. Its Witten index is clearly zero due to the 2ℓ zero-modes Ψ^I_0 of the internal fermionic coordinates ($I = 1, \ldots, 2\ell$). In order to get a non-trivial contribution from this sector one should compute anomalous amplitudes involving insertions in the internal SCFT such that all fermionic zero-modes Ψ^I_0 are soaked up. The contribution is now measured by non-vanishing generalized Witten indices

$\widehat{\mathcal{I}}_0 = \mathrm{Tr}\, F^\ell (-)^F$.[7] Irrespective of the dimension, anomalous amplitudes descending from the $D = 10$ exagon meet the requirements. The string computation is clearly the same as in ten dimensions but the interpretation in terms of the lower dimensional field theory is different. With respect to the canonical anomaly, the anomalous amplitudes under consideration are associated to higher derivative CP-odd terms involving scalars and KK vectors. These receive anomalous contributions not only from loops of massless fermions and but also from their higher KK-modes. The presence of such anomalous terms reflects the presence of a non-trivial coupling of the universal RR 2-form that compensates for the anomalous variation of the fermion determinants only when $N = 32$ as in $D = 10$, i.e. only when the RR tadpole of the identity sector with $\mathcal{I}_0 = 0$, is enforced. If one turns on continuous Wilson lines, thus generically breaking SO(32) to $\mathrm{U}(1)^{16}$, one is effectively trading some anomalous contribution of the massless fermions to the CP-odd thresholds with an additional contribution of the massive BPS states. If one turns on non-commuting Wilson lines, equivalent to a quantized background for the NS-NS antisymmetric tensor $B_{\text{NS-NS}}$, one is reducing both the rank of the Chan-Paton group as well as the number of gaugini and the coupling of the universal RR 2-form. Notice that this has a neat counterpart in the dual heterotic string. In this case, modular invariance replaces the RR tadpole condition in ensuring that any anomalous variation of the one-loop effective action is zero. When a gauge symmetry G in the heterotic string is realized in terms of a world-sheet current algebra at level κ the effective GS coupling is rescaled by that same factor. This is precisely what happens in the type-I settings we have just discussed if one identifies the level of the worldsheet current algebra κ with $2^{r/2}$ where r is the rank of $B_{\text{NS-NS}}$.

In non-trivial open string configurations involving sectors with vanishing Witten index $\mathcal{I}_i = 0$, other than the identity sector, one can apply similar considerations. The vanishing of $\mathcal{I}_i$ signals the presence of a certain number d of unsoaked fermionic zero-modes together with their bosonic superpartner under worldsheet supersymmetry. This essentially free piece of the internal SCFT is more like the T^d-toroidal compactification we have discussed so far. Indeed one can trace the different higher dimensional origins of the relevant anomalous amplitudes by their different scalings with the internal "volumes". By volume here we do not neccesarily mean a geometrical one but simply the result of the integration over the compact bosonic zero modes X_0^I $(I = 1, \ldots, d)$. In the context of geometric orbifolds, different amplitudes[8] are in general associated to different fixed tori. The different scalings of the anomalous amplitudes allow one to identify the constraints on the different sets of branes and the corresponding RR tadpoles in sectors with $\mathcal{I}_i = 0$.

It is instructive to explore the implications of similar anomalous diagrams in different string settings. In vacuum configurations for the heterotic string, moduli

[7] The "quasi" topological index $\mathcal{F} = \mathrm{Tr}\, F(-)^F$ was introduced in the $N = 2$ context in [21].

[8] In our approach, amplitudes are linear combinations of characters. The former follow from the latter by inverting (2.7).

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fields such as the radii of toroidal compactifications and the blowing up modes of an orbifold specify non-linear σ-models with continuous non-compact symmetries. These continuous symmetries combine geometric transformations such as internal diffeomorphisms with non-geometric symmetries that arise because of the extended nature of the fundamental constituents. The related presence of winding states breaks the continuous symmetries down to discrete ones ("T-duality"). Some of these discrete symmetries of the tree-level effective lagrangian are broken by quantum effects. On the one hand, they act by chiral transformations on the fermions that transform with moduli-dependent phases. The discrete charges (modular weights) are very sensitive to the sector each massless fermion belongs to. On the other hand, massive string states give rise to moduli-dependent thresholds corrections to CP-odd terms, related to the anomalous amplitudes discussed above, in theories with fixed tori, i.e. internal sectors with $\mathcal{I}_i = 0$. In $\mathcal{N} = 1$ supersymmetric compactifications, these CP-odd threshold corrections are anomalous in that they violate an integrability condition [24]. This violation is not renormalized beyond one-loop [25]. The combined variation of the contribution of the massless fermions and the CP-odd thresholds under discrete T-duality transformations is non-zero. There is howevere a universal GS counterterm that cancels the left-over discrete anomaly [24]. World-sheet modular invariance at one-loop thus guarantees the absence of target space modular anomalies.

After inclusion of all stable bound-states of strings and branes "T-duality" is expected to be promoted to "U-duality". In theories with open and unoriented strings, T-duality is broken at the perturbative level by the presence of the background brane configuration. Still, by including the contribution of branes wrapping around cycles of the internal manifold one expects to recover a discrete T-duality symmetry isomorphic to a T-duality subgroup of the U-duality of the parent theory. Absence of T-duality anomalies would then be a consequence of the vanishing of RR-tadpoles, including sectors with $\mathcal{I} = 0$, much in the same way as absence of T-duality anomalies is a consequence of modular invariance in closed-string theories.

Following the same line of reasoning, one may argue that the absence of discrete anomalies in non-supersymmetric vacua requires the absence of all massless RR tadpoles. Although very little is known about the effective lagrangian in this case we expect the anomalous terms including the anomalous CP-odd thresholds to be "topological" and as such to be non-renormalized beyond one loop except possibly by world-sheet instantons or D-istantons.

Let us conclude this section with a side remark on the odd-dimensional case. Although there are no local anomalies for odd dimensional manifolds without boundaries and/or branes, global anomalies cannot be a priori excluded [27]. It is reassuring to observe that these are absent for toroidal compactifications with a Chan-Paton group of even rank as required by RR tadpole cancellation, i.e. $N = 32 \times 2^{-r/2}$ or, for open descendants without open strings, $N = 0$.

6. Discussion and concluding remarks

Most of the vacuum configurations with open and unoriented strings are not simply geometric compactifications of ten-dimensional theories with open and unoriented superstrings. In a non-trivial background, genuine lower dimensional open string excitations can be located at new kinds of branes that might not admit a large volume description. Still one can try to extract some insights from the anomaly polynomial we have derived in some generality. In principle, once the topological data of the "compactification" manifold are identified, one can extract some useful information about the relevant brane configuration and/or the vacuum gauge bundle by comparing the WZ couplings found in the genuine string computation with the ones that would result from a naive compactification. All additional terms are to be related to extra bound-states of branes and/or to non-trivial configurations of the background gauge fields. We have parametrize the relevant information in terms of the boundary reflection coefficients B_a^i, i.e. one-point functions of massless states of type (i, i^C) on the disk with boundary conditions of type a. These B_a^i enjoy some interesting completeness properties and behave as vielbeins, i.e. they allow one to transform an index of type i labelling a sector of the closed string spectrum that flows in the transverse channel into an index of type a that labels the independent Chan-Paton multiplicities. It is conceivable that the B_a^i satisfy some interesting evolution equation that would allow one to compute them starting from special "rational" points in the moduli space of the vacuum configuration.

At these points, classifying all possible boundary conditions compatible with preserving half of the bulk symmetries is expected to be feasible. The worldsheet SCFT description pursued in this paper and the topological nature of the anomaly-related terms should be sufficient to show that the B_a^i for massless sectors with non-vanishing Witten index should be independent of the moduli. Indeed, for D-branes wrapped around supersymmetric cycles in Calabi-Yau spaces it is known that B_a^i for i a massless sector are quasi-topological [28]. For A-type boundary conditions the B_a^i, with i running over the (c, c) primary fields and a over the middle cohomology cycles, only depends on the complex structure deformations and may be computed at large volumes. For B-type boundary conditions the B_a^i, with i running over the (c, a) primary fields and a over the vertical cohomology cycles, only depends on the Kähler structure deformations and can be computed in terms of the "quantum intersection form", i.e. the "topological intersection form" (an integer) plus worldsheet instanton corrections. For massive sectors this is no longer expected to be the case, but the polynomial equations satisfied by the B_a^i may rescue the situation [15]. It would be interesting to find a way to extend the arguments of [28] based on the topological twist of $\mathcal{N} = 2$ SCFT to worldsheet theories that only enjoy $\mathcal{N} = 1$ superconformal symmetry.

We have not discussed the tadpoles in the NS-NS sector that may be disposed of by means of a Fischler-Susskind mechanism. The latter may destabilize the vacuum configuration. A signal could be found in behaviour of the B^i_a at points where a given brane configuration could decay into its constituents or tachyon condensation could trigger the formation of a stable non-BPS bound state.

The applications of the anomaly-tadpole correspondence we have established go far beyond of the contexts exploited in this paper. Indeed, our results only rely on the consistency between the open and closed string channel in the presence of boundaries and crosscaps. Relying on the $\mathcal{N} = (1,1)$ superconformal symmetry on the worldsheet enables one to isolate the odd spin-structure as the source of all potential anomalies. Modular invariance then allows one to identify anomalous amplitudes with RR tadpoles. Similar analyses can be applied to any (BPS or not) brane configuration satisfying the consistency requirements. Tadpoles are associated to closed string absortion processes encoded in the dynamics of boundaries and crosscaps on the worldsheet, Anomalies enter as one-loop CP-odd effects in the effective theory governing the dynamics of the brane configuration. The UV-IR dictionary relating these two effects is expected to be realized in any SYM/SUGRA correspondence described in terms of boundaries on the string worldsheet.

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